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RESEARCH DEPARTMENT OF EMPLOYMENT AND INTEGRATION

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Optimal admission to higher education

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Abstract

This paper constructs higher education admission rules that maximise graduation rates and thus increase the skill level of the work force. An application shows that students with a low level in mathematics in secondary school ought to find it easier to be admitted to an economics programme than to law or psychology programmes, even though economics is the most difficult programme from which to graduate without a strong background in mathematics. Indirect gains from optimal admission include the potential of making whole cohorts of students more able to graduate with a higher education degree.

Keywords: Skill level, work force, higher education

JEL Classification: I21

1 Introduction

High graduation rates and a smooth transition by students through the higher educational system are important elements for attaining a high skill level of the work force. Admission policy to higher education is a crucial factor for the allocation of students to different programmes and institutions, for graduation rates and for incentives in the secondary educational system.

University admission policies are under debate in many countries. The British debate includes the lack of applicants to university programmes that require a strong background in mathematics and science because secondary school ‘students choose “easier” subjects to get higher grades’ (Sunday Times, August 17, 2003, p. 6). One virtue of the admission system that this article proposes is that it can remove the inconsistency between the rational choices of secondary school students and the aim of the education system of providing students with a good background for obtaining a university degree.

This article develops guidelines for the construction of higher education admission criteria that maximise graduation rates. Such admission criteria are ‘optimal’ in the sense that deviations from them lead to unnecessarily low shares of students graduating from higher education. The empirical analysis of the paper demonstrates the application of the econometrics necessary for obtaining the parameters of interest for implementing optimal admission.

According to the theory in this article, introduction of optimal admission leads to increases in graduation rates that differ across programmes according to the variation in (1) the differences in initial graduation rates between various groups of students, (2) the strength of the relation between graduation rates and general ability to study, and (3) the composition of the intake of different groups of students. Adoption of optimal admission will have direct positive effects on the number of students graduating from higher education. In a formal analysis, I trace the indirect effects of the subject choices of secondary school students and the subsequent change in graduation rates in higher education.

The British literature includes Smith and Naylor (2001) and Johnes and McNabb (2004), who analyse attrition for a cohort of British undergraduate students. Determinants include degree subject and pre-entry characteristics such as grades and school types at the secondary level.

One strand of the American literature deals with the ability of the SAT (Scholastic Aptitude Test) to predict college admission (see Manski and Wise

(1983)) and freshman grades (see Barro (2001) and Rothstein (2004)). In combination with high school grades, the SAT score is used for determining admission to US colleges and universities. For example, the University of California constructs an admission index consisting of a weighted average of SAT scores and high school grades. If a student from a public secondary school in California has a score above a certain threshold on this index, this student is automatically admitted to the University of California system¹. The recent contributions by Bound, Lovenheim and Turner (2010) and Heckman and LaFontaine (2010) analyse graduation rates for whole cohorts of American youth. The problem of attrition in the American higher education system figures prominently in the educational and sociological literature, where the seminal contribution is Tinto (1993). However, the previous literature does not include the optimising approach to admission policies that this article proposes.

The social science programmes in the empirical analysis are 3-year structured programmes at the University of Copenhagen, leading to a bachelor degree in a specific field. These programmes thus follow the aims of the Bologna process with respect to increasing the transparency of education and facilitating recognition for further studies (see for example European Commission (2010)). The empirical results of this article might thus have relevance in many countries.

The empirical analysis yields ostensibly counter-intuitive results: only students with a strong background in mathematics ought to be admitted to a psychology programme, whilst students with less mathematics should find it easier to be admitted for studying economics rather than law. These rules are valid even though economics is the most difficult programme from which to graduate without a strong background in mathematics.

The contributions of this article are applicable at several levels of decision-making: for constructing admission rules for one programme at one university, for joint admission rules for several programmes in one university and for joint admission criteria for programmes across several universities.

A higher graduation rate from higher education, better preparation in secondary school for attending higher education and the consequential higher skill level of the work force might not be the only objectives for admission policy to higher education. Other goals might include access to higher ed-

¹See the ‘admission index’ at University of California, <http://www.universityofcalifornia.edu/admissions/>

ucation for groups of students with a variety of backgrounds. Such other goals might or might not conflict with the efficiency considerations in this article. To the extent that other goals are in conflict with efficiency, this article establishes the framework for calculating the costs of pursuing these goals in terms of lower graduation rates.

The article is organised as follows. Section 2 derives the optimal admission rules that maximise graduation rates in the higher education system. The section also derives an analytical expression for the approximate increase in graduation rates. Section 3 demonstrates how to estimate the parameters of interest in the construction of optimal admission rules and how to present the results succinctly. Optimal admission rules are derived for the social science programmes at the University of Copenhagen. Section 4 traces the impact of a change from non-optimal to optimal admission on the choice of subjects by students in secondary school. Section 5 analyses how this behavioural change alters the observed graduation rates for groups of secondary school students. The impact on graduation depends on the extent to which differences in graduation rates are due to self-selection or to a heterogeneous impact of subjects across students. Section 6 concludes.

2 Maximisation of graduation rates

This section contains the formal analysis of admission to higher education viewed as an optimising problem, with the goal of maximising graduation rates. First, the section proves the optimal admission rule. Second, the section calculates how much optimal admission changes thresholds compared to a system with a common threshold. Third, the section develops an expression for the increase in graduation rates.

Admission is conducted by fixing a threshold on an index of general ability to study as a minimum requirement for admission. As the grade point average (GPA) in secondary school is a natural component of such an index, I conduct the exposition under the assumption that grades in secondary school are the only component of this index. Students can attend several paths or types of school in the secondary school system before applying to higher education. Secondary school paths are denoted i , $i = 1, 2, \dots, n$. The threshold GPA for path i is denoted g_i , and the total amount of admitted students from path i becomes

$$\Lambda_i(g_i) = A_i \int_{g_i}^{g^{\max}} a_i(k) dk, \quad (1)$$

where g^{\max} is the maximum grade, $a_i(k)$ is the density function for the number of applicants with GPA of k , and A_i is the total amount of applicants with a secondary school examination from path i .

Graduation is denoted y , which takes the value 1 for graduating and 0 for not graduating. The probability of graduating for a student from path i with GPA k is denoted $p_i(k)$, that is

$$p_i(k) = P(y = 1|k, \text{path } i) = E(y|k, \text{path } i).$$

The first equal sign is the definition of the conditional graduation probability $p_i(k)$, whilst the second equal sign shows that the graduation probability is equal to the expected value of the variable y , conditional on grades and being in path i in secondary school. This section assumes that $p_i(k)$ increases in k , $\partial p_i(k) / \partial k > 0$. The case when higher grades in secondary school do not increase graduation probability is simple and is covered by example in the empirical section.

The expected number of students graduating from the programme becomes

$$K_i(g_i) = A_i \int_{g_i}^{g^{\max}} p_i(k) a_i(k) dk.$$

The number of students finishing the programme decreases in the admission threshold, whilst $K_i(g_i) / \Lambda_i(g_i)$, the share of students graduating from the programme, increases in the admission threshold.

The goal is to maximise the number of students graduating from the programme, $\sum_{i=1}^n K_i(g_i)$, given a certain number of admitted secondary school students

$$\Lambda = \sum_{i=1}^n \Lambda_i(g_i).$$

The solution must entail admission thresholds' not exceeding the maximum grade

$$g_i \leq g^{\max}, \quad i = 1, 2, \dots, n.$$

The Kuhn-Tucker stationarity conditions for maximisation become

$$\begin{aligned} \frac{\partial}{\partial g_i} \left[\sum_{i=1}^n K_i(g_i) \right] + \lambda \frac{\partial}{\partial g_i} \left[\sum_{i=1}^n \Lambda_i(g_i) \right] - \mu_i = \\ -A_i p_i(g_i) a_i(g_i) + \lambda A_i a_i(g_i) - \mu_i = 0, \quad i = 1, 2, \dots, n. \end{aligned} \quad (2)$$

The complementary slackness conditions are

$$\mu_i (g_i - g^{\max}) = 0, \quad i = 1, 2, \dots, n.$$

If $g_i = g^{\max}$, students from group i are not admitted to the programme. Interior solutions, $g_i < g^{\max}$, imply $\mu_i = 0$ for the admitted groups. Hence, for two groups admitted to the programme, group i and group j , a reformulation of (2) gives

$$\lambda = p_i(g_i) a_i(g_i) = p_j(g_j) a_j(g_j). \quad (3)$$

This is the rule for optimal admission to higher education. Threshold GPAs for groups of students should be set to equalise marginal graduation probabilities. The expected number of graduating students is maximised when the last admitted student from each of the groups has the same probability of graduating from the programme. The threshold values do not depend on the number of applicants, A_i , or on the distribution of students according to grades in secondary school, $a_i(g_i)$.

Figure 1 contains a stylised illustration of the optimal admission rule in the case of two groups of students, group i and group j . The conditional graduation probability is assumed linear, with the same slope for both groups.

Figure 1 around here

The admission system with a common threshold GPA value \bar{g} is illustrated by the dotted vertical line denoted ‘Not optimal’. The difference in graduation probability between the two groups corresponds to the distance $p_i(\bar{g}) - p_j(\bar{g})$. Transition to the optimal admission system involves a move from $p_i(\bar{g})$ to $p_i(g_i)$ for group i students and a move from $p_j(\bar{g})$ to $p_j(g_j)$ for group j students. On the horizontal dotted line, the marginal graduation probability is the same for the two groups, $p_i(g_i) = p_j(g_j)$, and this admission rule is optimal in the sense that no further improvement in the aggregate

graduation rate is possible for a given number of admitted students. Optimal admission to higher education is horizontal, not vertical.

The second step in the analysis is to obtain the differences in admission thresholds in terms of the parameters of the problem, the slopes of the conditional graduation functions, and the difference in initial graduation probabilities. A Taylor expansion of the conditional graduation probability function around \bar{g} gives

$$p_i(g_i) = p_i(\bar{g}) + p'_i(\bar{g}_i)(g_i - \bar{g}), \quad i = 1, 2, \dots, n, \quad (4)$$

where $p'_i(\bar{g}_i)$ is the slope of the conditional graduation probability function for students from path i evaluated at GPA level \bar{g}_i , which lies between g_i and \bar{g} . Furthermore, assume a constant slope in the relevant range of grades, $p'_i(\bar{g}_i) = p'_i$. For another group of students j assume a constant slope in graduation probability, $p'_j(\bar{g}_j) = p'_j$, which differ from the slope of group i by a constant, $p'_j = p'_i + \pi_{ji}$.

These assumptions about the slopes for group i and j in combination with the optimality condition (3) yields the difference in admission thresholds

$$g_j - g_i = [(p_i(\bar{g}) - p_j(\bar{g})) - (g_j - \bar{g})\pi_{ji}] / p'_i. \quad (5)$$

The first part of the right hand side, $(p_i(\bar{g}) - p_j(\bar{g})) / p'_i$, is the difference in admission thresholds if group i and group j have the common slope p'_i . If the graduation probability for group i is higher than that for group j , evaluated at the common threshold \bar{g} , $p_i(\bar{g}) > p_j(\bar{g})$, the threshold GPA for group j has to be higher than the threshold GPA for group i , $g_j > g_i$. The larger the difference in graduation between the two groups at the common threshold, the larger the difference in the threshold GPAs will be. When the denominator p'_i is large, that is, when a large difference exists in graduation rates between students with high and low GPAs, a small difference between the threshold GPAs among the two groups will equalise the marginal graduation rates.

The second part of the right-hand side of (5), $(g_j - \bar{g})\pi_{ji}/p'_i$, adjusts the difference in threshold GPAs for the impact of different slopes in conditional graduation probability. If group j has a larger slope than group i , $\pi_{ji} > 0$, the difference in threshold GPAs $g_j - g_i$ is reduced, whilst $\pi_{ji} < 0$ implies a larger difference in threshold GPAs relative to the case with a common slope.

In the case of a common slope, I have obtained the difference in admission grades between groups of students but not the change from the common

threshold $g_i - \bar{g}$. As Figure 1 illustrates, I have determined the magnitude of the horizontal line from $p_i(g_i)$ to $p_j(g_j)$ but not the height or location of this line segment. This location depends on the relative number of students in the groups and is to be determined.

When an admission threshold is lowered, the number of new students depends on the distribution of applicants, which is unknown. The following calculations apply the simplifying assumption that the densities are constant in the relevant ranges of GPAs, $a_i(k) = a_i$ (this amounts to a zero order approximation of the true, unknown distribution by a local uniform distribution function). Calculation on (1) yields the following expression for the change in the number of admitted students

$$\Delta\Lambda_i(g_i) = -A_i a_i \Delta g_i, \quad \Delta g_i = g_i - \bar{g}.$$

As $\sum_{i=1}^n \Delta\Lambda_i(g_i) = 0$, the change in admission threshold in group j becomes

$$\Delta g_j = g_j - \bar{g} = \sum_{i=1}^n s_i (g_j - g_i), \quad (6)$$

where

$$s_i = A_i a_i / \sum_{i=1}^n A_i a_i, \quad \sum_{i=1}^n s_i = 1.$$

As a_i is the density of applicants and A_i the number of applicants, s_i is thus the share of group i students among the applicants.

Expression (6) shows that the admission threshold for each group of students can be calculated from the share of students of the different groups and the differences in admission thresholds from expression (5). In the case of common slopes, the change in admission threshold appears directly on the left hand side of (6). When the slopes differ for groups of students, expression (5) inserted in (6) yields an equation with the solution

$$\Delta g_j = g_j - \bar{g} = \sum_{i=1}^n s_i \{ [p_i(\bar{g}) - p_j(\bar{g})] / p'_i \} / \left(1 + \sum_{i=1}^n s_i \pi_{ji} / p'_i \right). \quad (7)$$

In Figure 1 a large share of group i students will result in a large increase in threshold GPA for group j students, and correspondingly a small decrease in threshold GPA for group i students. The horizontal distance from g_i to g_j in Figure 1 is divided into two line segments corresponding to the share

of the two groups in the pool of applicants, and this division determines the height of the dotted vertical line denoted ‘Optimal’.

The final step is to assess the magnitude of the gain in graduation rates by an application of the optimal admission rule. I deduct an analytical expression for the approximate gain under the previously applied simplifying assumption of constant densities, $a_i(k) = a_i$, as in (6) and constant and identical slopes of the graduation probability functions $p'_i = p'_j = p'$ (when the slopes differ, a sensitivity analysis is performed by inserting alternative values). Furthermore, assume without loss of generality that the grade distribution is centred at 0, that is, $\bar{g} = 0$.

With these assumptions the change in the number of group i students graduating from the programme becomes

$$\Delta K_i(g_i) = A_i a_i \int_{g_i}^0 p_i(k) dk,$$

which in normalised form becomes

$$\Delta \kappa_i(g_i) = \Delta K_i(g_i) / \sum_{i=1}^n A_i a_i = s_i \int_{g_i}^0 p_i(k) dk. \quad (8)$$

As the number of admitted students does not change, introduction of optimal admission implies an increase in the graduation rate K/Λ on

$$\Delta K/\Lambda = \left(\sum_{i=1}^n A_i a_i / \Lambda \right) \sum_{i=1}^n \Delta \kappa_i(g_i) \quad (9)$$

The increase in the graduation rate is thus proportional to the sum of the normalised changes in graduation $\sum_{i=1}^n \Delta \kappa_i(g_i)$, where the scaling factor is $\sum_{i=1}^n A_i a_i / \Lambda$.

Insertion of (4) in (8) gives

$$\Delta \kappa_i(g_i) = -s_i p_i(0) g_i - \frac{1}{2} s_i p'_i g_i^2. \quad (10)$$

A decrease in the threshold, $g_i < 0$, implies that more students from the group enter and graduate from the programme. The first term on the right-hand side is positive and corresponds to a graduation probability for all new admitted students on $p_i(0)$. The second term is negative, taking into

account that the graduation probability for the marginal student decreases when the threshold is lowered. In Figure 1 the first term corresponds to a rectangle with base $g_i\bar{g}$ and height $p_i(\bar{g})$, whilst the second term corresponds to a triangle with base $g_i\bar{g}$ and height $p'g_i\bar{g}$ (situated at the location of the downward sloping arrow). When fewer students are admitted, $g_i > 0$, the interpretation of (10) is analogous.

I add (10) for the different groups of applicants, apply (6) and get the following expression for the change in graduation for the programme

$$\sum_{i=1}^n \Delta\kappa_i(g_i) = - \sum_{i=1}^n s_i p_i(0) \sum_{j=1}^n s_j (g_i - g_j) - \frac{1}{2} p' \sum_{i=1}^n s_i g_i^2. \quad (11)$$

The first term on the right-hand side of (11) is the sum of the main effects, whilst the last term is the sum of the terms that corrects for the fact that not all students involved in the exchange between the groups have initial graduation probabilities.

Inserting (5) into (11) yields the expression for the increase in the graduation rate for the programme in terms of the parameters of the problem. The case of $n = 3$ will suffice for the purpose of analytical insight and the use of this insight in applications ($n = 3$ corresponds to the number of groups in the empirical section that follows).

The result for the gain in graduation rate for the programme is

$$\begin{aligned} \sum_{i=1}^3 \Delta\kappa_i(g_i) = & \left\{ [s_1 s_2 (s_1 + s_2) d_{21}^2 + s_2 s_3 (s_2 + s_3) d_{32}^2 + s_1 s_3 (s_1 + s_3) d_{31}^2] / 2p' \right\} \\ & + s_1 s_2 s_3 (d_{21} d_{23} + d_{32} d_{31} + d_{31} d_{21}) / p', \end{aligned} \quad (12)$$

where $d_{21} = p_2(0) - p_1(0)$, $d_{32} = p_3(0) - p_2(0)$ and $d_{31} = p_3(0) - p_1(0)$.

The gain in graduation rate is high when the initial differences in graduation rates between the groups are large (large numerical values of $p_2(0) - p_1(0)$, $p_3(0) - p_1(0)$ and $p_3(0) - p_2(0)$). A high increase in the graduation rate as GPA increases (high p') implies a small gain in the aggregate graduation rate, whilst a small value of this slope implies that many students with low graduation rates are replaced by students with high graduation rates. Reshuffling between two groups gives the highest gain in graduation rate when the two groups are of equal size (e.g. $s_1 = s_2$), as equal group size implies that many students with low graduation rates are replaced by many students with high graduation rates.

Expression (12) for the gain in the aggregate graduation rate applies to one programme. Programmes will differ in increases in graduation rates according to the variation in (1) differences in initial graduation rates between various groups of students, (2) the strength of the relation between graduation rates and GPA and (3) the composition of the intake of students from different groups.

3 Empirical analysis of optimal admission

This section illustrates the application of the rules developed in the previous section. The necessary information is retrieved from a data set with information for students admitted to various university programmes and applied for constructing optimal admission rules and approximate gains in graduation rates.

According to the optimal admission rule, expression (5), two pieces of information are sufficient for calculating the difference in GPA between different groups in the case of a common slope: (a) the difference in graduation probability at the common threshold and (b) the slope of the graduation probability with respect to grades. Furthermore, to identify the location of the differentiated threshold according to expression (6), we need information about (c), the local share of the groups of students. According to expression (12) this information is also sufficient for calculating the approximate gain in graduation rates.

The information for constructing optimal admission rules is estimated on data for students admitted to the four largest bachelor programmes in the social sciences at the University of Copenhagen. The following description of the Danish education system provides the background for understanding the results.

A bachelor student is expected to graduate after 3 years of study. The success criteria in this article is graduation within 4 years of study.

The Danish secondary school system is in essentially a two-tiered system in which about 40% enter apprenticeship training (similar to that of Germany), whilst the majority of the remaining 60% enter grammar schools ('gymnasium') that are a prerequisite for admission to higher education (see Albæk (2009) for an overview of the Danish apprenticeship system). The major and traditional grammar school path is the 'common' secondary school, which lasts three years. Upon entrance to the common secondary school,

students choose between the ‘mathematics branch’ (with emphasis on mathematics and science) and the ‘language branch’ (with emphasis on languages).

As Table 1 shows, the largest share (46%) of university students in the four social sciences programmes come from the mathematical branch of the common secondary school whilst 25% come from the language branch. The remaining 30% come from various types of specialised grammar schools. For the present purpose this last group of students is categorised as ‘other’.

Table 1 around here

Table 1 shows that the average graduation rate for all students in the social sciences was 55%. This rate is higher than the figures presented for the US in Bound, Lovenheim and Turner (2010) and for Norway in Hovdhaugen (2009). Students who do not graduate consist of both dropouts from higher education and students who transfer to another programme.

Students from the common secondary schools have higher graduation rates than students from the ‘other’ group: the difference is a significant 21 percentage points. In total no significant difference in graduation rates appears between the mathematics and the language groups. However, when significant differences exist between the mathematics and the language groups in the four individual programmes, the mathematics students have higher graduation rates. The low aggregate graduation rate for mathematics students is partly a consequence of the low graduation rate in the economics programme, which attracts a substantial share of all mathematics students.

Table 1 orders the programmes according to magnitude. The law programme is most important in this respect, as 47% of the students enter this programme, whilst 12% enter the psychology programme.

Figure 2 shows the grade distribution for the four programmes: the top panel displays the distribution of the three secondary school branches for each of the programmes. The main inference drawn is that only minor differences appear in grade distribution amongst the three groups in any of the programmes.

Figure 2 around here

The secondary school grades of the students in the data are standardised by withdrawing the mean of the grades and dividing by the standard deviation for the population of Danish secondary school students.² The Ministry of Education attempts to make grades comparable across secondary schools. Written exams from the Ministry are mandatory in all secondary schools, and grading is monitored by the Ministry. Oral exams are conducted by the teacher and an external moderator.

In the bottom panel of Figure 2, the three groups are combined, showing that economics students have lower grades than those admitted to the law programme; the law students in turn have lower grades than those admitted to political science and psychology. Most students are admitted to the various programmes solely on the basis of their secondary school GPAs. The threshold GPAs varied between 0 for the economics programme to nearly 2 for psychology. As the grade distribution for the population of Danish secondary school students is approximately normal, only a tiny fraction of secondary school students is eligible for social sciences programmes outside of economics and law. However, a minority of varying magnitude (10-30%) are admitted on the basis of supplementary criteria (e.g. work experience, volunteer work). These admissions imply that data exists not only for students with GPAs above the acceptance threshold but also for those with GPAs below it.

Figure 3 presents the estimates of expected probability of graduating, conditional on secondary school GPA. The figure shows the expected graduation probability for the three secondary school branches for each of the four programmes. These conditional expectation functions are highly non-linear, making non-parametric estimation adequate.

Figure 3 around here

The lines are predicted graduation rates from local polynomial regressions. The degree of the polynomial is set at two, which is the minimum for estimating the slopes of the conditional expectation functions. The slopes are necessary components in the assessment of optimal admission (see section 2). Kernels are Gaussian. The bandwidth is set at 0.62 for the largest group

²The grading scale in the sampling period was the ‘13-scale’, which ranges from 0 to 13. The average GPA for all students in the common high school was approximately 8, and the standard deviation was about 1.

of students, mathematics students in the law programme. The bandwidths for the remaining groups are increased according to the asymptotics of the plug-in bandwidth, which minimise the mean square error of the regression (see Wand and Jones (1995), p. 139). All data points enter the estimations, including the tails of the grade distribution, where data are slim and inference imprecise. Figure 3 shows the results for GPAs from -1 to 2.5, the relevant range for admission policy.

In the three largest programmes – law, economics and political science – the graduation probability increases with secondary school GPA for most branches of students (the exceptions are mathematics students in economics with very high GPAs and other students in political science with GPAs above 1). In psychology, graduation probabilities are constant for mathematics students with GPAs above 0, and for language students with GPAs above 1.³

Optimal admission rules appear in Figure 3 as horizontal lines. For the law programme, the marginal graduation probability for the groups is set at 0.60. The graduation probability for other students is below the horizontal line for all levels of GPA. Consequently, this group should not be admitted to the law programme. The height of the horizontal line is found as follows: when the group of other students does not enter the programme, the intake of mathematics and language students increases to ensure an unaltered number of students in the programme. This increase takes place by lowering the common threshold for mathematics and language students to 0.5. The values of the graduation probabilities in Figure 3 appear in Table 1: 0.649 for mathematics students and 0.586 for language students. The slopes of the curves in Figure 3 appear in Figure 4, and for the law programme the slopes at GPA level 0.5 also appear in Table 1: 0.090 for the mathematics and 0.040 for the language group. I insert in expression (7) the information about the slopes, the shares of the two groups and the difference in graduation probability, and I thus obtain the threshold changes that appear in Table 1:

³If no students below the common admission thresholds were admitted, the conditional graduation functions would be truncated at the common threshold. In such a case extrapolation of the conditional graduation functions below the threshold is necessary for constructing optimal admission thresholds. Graduation probabilities below the common thresholds in Figure 3 are not estimates of population parameters but are contingent upon the policy determining admission of students with GPAs below the common threshold. To the extent that institutions are able to identify students with a high probability of graduating, contingent on their GPA, the graduation probabilities below the common threshold in Figure 3 are higher than the average among the applicants.

a decrease in the threshold for mathematics students of 0.5 and an increase for language students of 0.5.⁴ The graduation probability is 0.6 for both mathematics students with a GPA level of 0 and for language students with a GPA level of 1 according to Figure 3, and the optimal admission rule for the law programme is thus shown as a horizontal line with the height of 0.6.

Figure 4 around here

The gain in graduation rate for the law programme is assessed in three steps: an increase when the university admits only the previous amount of students from the common secondary school (5.9 percentage points), a decrease when the admission threshold is lowered and more common secondary school students are admitted (-0.3 percentage points), and an increase in graduation rate when a differentiated threshold equalises the marginal graduation rate between mathematics and language students (0.7 percentage points), obtained from (9) and (12). For the programmes analysed in this article, I assess the correction factor in (9) to 0.5 (on the basis of simulations). The result is the increase in graduation rate of 6.2 percentage points shown in Table 1 corresponding to an increase of 10.4% in the number of graduating law students (the 0.065 divided by the previous average graduation rate on 0.598).

For the economics programme, all three groups of students are admitted after the change from a common threshold to optimal admission. The common threshold is 0.0, and for each of the three groups Table 1 lists the estimated graduation probabilities in Figure 3 and the corresponding slopes of the conditional graduation functions in Figure 4. Insertion of these values in (7) gives the change in admission threshold listed in Table 1: -0.2 for mathematics students, 0.1 for language students and 0.6 for other students.

The optimal difference in admission thresholds between mathematics and language students is thus 0.3 for the economics programme, which is smaller than the threshold difference of 1.0 between mathematics and language students for the law programme. It should thus be easier for language students to be admitted to the economics programme than to the law programme.

⁴The top panel of Figure 2 shows that the density functions are approximately identical for the three groups of admitted students. I assume that the (unobserved) distributions of the applicants are also identical and that the shares of admitted students in Table 1 apply in the calculations.

This finding is valid despite language students in the relevant ranges of GPA having more difficulty graduating in economics than in law (as the vertical distance between the conditional graduation functions in Figure 3 makes clear).⁵ The reason is that the slopes of the conditional graduation functions are substantially higher in the economics programme than in the law programme in the relevant GPA ranges, which both Figure 4 and the values of the slopes at the common thresholds in Table 1 make clear.

For the economics programme the gain in the aggregate graduation rate from adopting optimal admission is modest. Application of the approximation (12) yields an increase in the aggregate graduation rate of 0.7 percentage points when the slope for the other group enters the calculation (a sensitivity check yields 0.5 percentage points when inserting the larger slope for mathematics group, whilst the smaller slope for the language group gives 1.0 percentage points). The main reason for the small gain is the high association between grades and graduation probability in the economics programme, implying a small reallocation between the groups before graduation rates for the marginal students are equalised. As the initial graduation rate is 0.401 for the economics programme, the expected increase in the number of graduating students is 1.8%.

The political science programme has a common threshold for mathematics and language students, whilst no students from the ‘other’ group are admitted. The increase in graduation rate is estimated at 6.8 percentage points.

For the psychology programme, the horizontal line corresponds to a graduation probability of 0.67. Irrespective of GPA, mathematics students have graduation probabilities above the line, whilst language students and students from the ‘other’ group have graduation probabilities on or below the line. As mathematics students have higher graduation rates than the two other groups irrespective of GPA, admitting only mathematics students maximises the graduation rate for the psychology programme. This finding proves valid despite the comparatively low advantage of being a mathematics student in the psychology programme: the difference in graduation probability is about 5 percentage points in the GPA range 1-2.5.

The highest increase in graduation is from changes in admission to the

⁵At a GPA level of 1.0, the admission threshold for language students in the law programme, language students have a 9% lower graduation probability than mathematics students in the law programme but an 18% lower graduation probability in the economics programme.

psychology programme. In this programme higher grades do not lead to a higher graduation probability, so expression (12) for the gain in graduation rate is not valid (as $p' = 0$). However, simple calculation shows that the gain in terms of increase in aggregate graduation rate is a substantial 13.6 percentage points.⁶ As the initial graduation rate was 55.9%, introduction of optimal admission is expected to increase the graduation rate of psychology students by 24.3%.

The difference in graduation rates between mathematics students and language students is statistically significant for the law, economics and psychology programmes (see the standard errors in table 1). Figure 3 shows small differences in graduation rates in the lower part of the grade distributions for both the law and the economics programme, and thus the larger differences in graduation rates in the upper part are significant.

For the four social science programmes in total, the gain in graduation from a transition to optimal admission is substantial. The estimated increase in the number of graduating students is a sizeable 10.3%.

This section has illustrated how to maximise graduation rates in higher education programmes by admitting more students with a high graduation probability and fewer students with a low graduation probability. Results show that mathematics students ought to have admission preference for the law, economics and psychology programmes. Such an admission structure could have derived consequences for students' choices of subjects in secondary school.

4 The choice of subjects in secondary school

This section establishes a framework for analysing the choice of subjects by secondary school students. Within this framework I trace the impact of a change in admission rules on the choice of subjects.

Assume that secondary school students choose amongst two subjects: mathematics leading to an A-level in mathematics and an alternative subject leading to a B-level in mathematics. The indirect utility for student i from following mathematics is specified as

$$U_A^i = \omega\gamma_A^i + w_i\delta_A + \epsilon_A^i, \quad (13)$$

⁶The statistics for graduation rates and shares in Table 1 enter the calculation as $0.287 \times (0.695 - 0.621) + 0.472 \times (0.695 - 0.452) = 0.136$.

where U_A^i is the utility from following mathematics, γ_A^i is the expected grade in mathematics, ω is the weight of mathematics in calculating the GPA used for admitting students to university programmes, w_i is a vector of variables determining the preferences for mathematics with the associated parameter vector δ_A , and ϵ_A^i is a term that includes preferences for mathematics beyond the expected grade and explanatory variables.

Correspondingly, the utility from attending the alternative non-mathematics subject leading to a B-level in mathematics, is specified as

$$U_B^i = (1 - \omega) \gamma_B^i + w_i \delta_B + \epsilon_B^i,$$

where U_B^i is the utility from attending the subject, γ_B^i is the expected grade in the non-mathematics subject, $(1 - \omega)$ is the weight of the non-mathematics subject, δ_B is a parameter vector associated with the explanatory variables and ϵ_B^i is the error term that includes preferences for the non-mathematics subject.

Mathematics is chosen by the student if

$$\omega \gamma_A^i - (1 - \omega) \gamma_B^i + w_i (\delta_A - \delta_B) > \epsilon_B^i - \epsilon_A^i.$$

The variance of the error term is

$$\sigma^2 = \sigma_{\epsilon_B}^2 + \sigma_{\epsilon_A}^2 - 2\sigma_{\epsilon_B \epsilon_A}$$

and normalising by the standard deviation, the following notation is adopted

$$\begin{aligned} \epsilon_i &= \frac{\epsilon_B^i - \epsilon_A^i}{\sigma} \\ z_i &= \frac{\omega \gamma_A^i - (1 - \omega) \gamma_B^i}{\sigma} + \frac{w_i (\delta_A - \delta_B)}{\sigma}. \end{aligned} \tag{14}$$

The probability that student i chooses mathematics thus becomes

$$P(\epsilon_i < z_i) = G(z_i),$$

where G is the distribution function of the random variable ϵ_i .

Differentiating yields

$$\frac{\partial G}{\partial \omega} = g(z_i) \frac{\partial z_i}{\partial \omega} > 0, \quad \frac{\partial z_i}{\partial \omega} = \frac{\gamma_A^i - \gamma_B^i}{\sigma} > 0,$$

where g is the density function. If the weight of mathematics in the admission criterion is increased, more students will choose this subject.

Three main assumptions drive the proof. First, students choose subjects by comparing the gain or utility of choosing mathematics with the utility of choosing some alternative subject. Second, the expected grade is included in the gain or utility of the choices. Third, a spread exists in the student preferences for choosing mathematics or the alternative. If these assumptions are fulfilled, an increased weight to mathematics in the calculation of the GPA will move students on the borderline of choosing this subject, whilst students with low relative preferences for mathematics will continue to study the alternative subject.

5 Behavioural change and changes in graduation rates

Optimal admission has the potential of increasing graduation rates from higher education. Giving more weight to mathematics in admission rules is expected to make more students choose mathematics. This increase of students studying mathematics might change the average graduation rates among both the mathematics and the non-mathematics students. This section investigates the impact of the policy change on the shift in graduation rates. To explain graduation probability, I present a framework that includes both self-selection and heterogeneous return to mathematics. Then I trace the effect of the policy change on expected graduation within this framework.

The graduation probability for student i in a particular programme is denoted y_i , which takes the value 1 for graduating and 0 for not graduating. The graduation probability is approximated by the linear probability model

$$\begin{aligned} y_i &= X_i\beta + \alpha_i m_i + e_i \\ \alpha_i &= a_0 + a_i, \end{aligned}$$

where X_i is a vector of graduation determinants, including grades, β is the associated coefficient vector, m_i is an indicator variable taking value 1 if mathematics is chosen and 0 otherwise, α_i measures the change in graduation probability that student i obtains as a consequence of studying mathematics, and e_i is the error term.

The term a_0 is the average change in graduation probability amongst the population of students and a_i is the deviation for student i from the

population average. The heterogeneous return to mathematics is specified as a random coefficient model, which is applied in the recent literature on the impact of schooling and training on labour market outcomes (see Card (1999) and Heckman et al. (1999)). Errors are assumed multivariate normal as in the seminal contribution on the random coefficient model by Björklund and Moffitt (1987).

The expected graduation probability for a student who has taken mathematics is

$$E(y_i|X_i, m_i = 1) = X_i\beta + a_0 + E(\alpha_i + e_i|\epsilon_i < z_i),$$

where ϵ_i and z_i are defined in (14).

Applying the expression for a standard normal variable truncated from above, the expected value of the conditional error term becomes

$$E(\alpha_i + e_i|\epsilon_i < z_i) = -\sigma_{a+e,\epsilon} \frac{g(z_i)}{G(z_i)}, \quad (15)$$

where G and g are the distribution and density functions from the previous section of the article, now assumed to follow the standard normal distribution. Covariances are

$$\begin{aligned} \sigma_{a+e,\epsilon} &= \sigma_{a\epsilon} + \sigma_{e\epsilon} \\ \sigma_{a\epsilon} &= \frac{\sigma_{a\epsilon_B} - \sigma_{a\epsilon_A}}{\sigma} \\ \sigma_{e\epsilon} &= \frac{\sigma_{e\epsilon_B} - \sigma_{e\epsilon_A}}{\sigma}. \end{aligned} \quad (16)$$

The expected graduation probability for a student who has not taken mathematics is

$$E(y_i|X_i, m_i = 0) = X_i\beta + E(e_i|\epsilon_i > z_i).$$

Applying the expression for a standard normal variable truncated from below, the expected value of the conditional error term becomes

$$E(e_i|\epsilon_i > z_i) = \sigma_{e,\epsilon} \frac{g(z_i)}{1 - G(z_i)}. \quad (17)$$

The difference in expected graduation probabilities between the mathematics and the non-mathematics students becomes

$$\begin{aligned}
& E(y_i|X_i, m_i = 1) - E(y_i|X_i, m_i = 0) \\
&= a_0 - \sigma_{ae} \frac{g(z_i)}{G(z_i)} - \sigma_{e,\epsilon} \left[\frac{g(z_i)}{G(z_i)} + \frac{g(z_i)}{1 - G(z_i)} \right].
\end{aligned} \tag{18}$$

For the law, economics and psychology programmes at the University of Copenhagen, mathematics students have a higher graduation probability than non-mathematics students conditional on GPA (see fig. 3) and the difference (18) is thus positive.

Each of the three terms of the right-hand side of (18) can contribute to a positive difference in graduation probability between mathematics and non-mathematics students. The first term contributes if the average student increases graduation probability by taking mathematics ($a_0 > 0$). The second term contributes to a positive difference if and only if (see (16))

$$\sigma_{ae} < 0 \Leftrightarrow \sigma_{aeB} < \sigma_{aeA}.$$

This inequality arises when students with high preferences for mathematics experience higher increases in graduating from the programme by studying mathematics in secondary school than do students with low preferences for mathematics. The third term on the right-hand side (18) contributes to a positive difference if and only if

$$\sigma_{e,\epsilon} < 0 \Leftrightarrow \sigma_{e\epsilon B} < \sigma_{e\epsilon A}.$$

This case applies when students with high preferences for mathematics have a higher unobserved probability of graduating from the programme than students with low preferences for mathematics (relative to the non-mathematics subject).

After establishing the framework, I now turn to the consequences of a policy change that gives mathematics a higher weight in the admission criterion.

First, consider the term capturing the heterogeneity of students taking mathematics. As $\partial g(z_i) / \partial z_i = -z_i g(z_i)$, differentiation of (15) yields

$$\frac{\partial}{\partial \omega} \left[-\sigma_{a+e,\epsilon} \frac{g(z_i)}{G(z_i)} \right] = (\sigma_{ae} + \sigma_{e\epsilon}) \left(z_i + \frac{g(z_i)}{G(z_i)} \right) \frac{g(z_i)}{G(z_i)} \frac{\partial z_i}{\partial \omega}.$$

For $z_i > -g(z_i) / G(z_i)$ the entity in the parenthesis on the right-hand side is positive, and calculation shows that this expression is positive for all values of z_i .

Given that heterogeneous return from mathematics contributes making the right-hand side of (18) positive ($\sigma_{ae} < 0$), the graduation probability for mathematics students decreases whilst the graduation probability of the non-mathematics students remains unaltered. Whilst the policy change induces more students to take mathematics, these students on average derive less from mathematics with respect to increases in graduation probability than those who took this subject before the policy change. In Figure 1, the upper curve moves downwards whilst the lower curve is unchanged.

Given that self-selection contributes to a positive difference in graduation rates between mathematics and non-mathematics students ($\sigma_{ee} < 0$), the graduation probability for students taking mathematics decreases. Whilst the policy change induces more students to take mathematics, these students on average have less preference for mathematics and do not have the same graduation probability in the programme as those who took the subject before the policy change. The upper curve in Figure 1 moves downwards.

The effect of the policy change on graduation for students who do not take mathematics becomes

$$\frac{\partial}{\partial \omega} \left[\sigma_{e,\epsilon} \frac{g(z_i)}{1 - G(z_i)} \right] = \sigma_{e,\epsilon} \left(-z_i + \frac{g(z_i)}{1 - G(z_i)} \right) \frac{g(z_i)}{1 - G(z_i)} \frac{\partial z_i}{\partial \omega}.$$

For $z_i < g(z_i) / (1 - G(z_i))$ the entity in the parenthesis on the right-hand side is positive, and calculation shows that this expression is positive for all values of z_i . Under the assumption of $\sigma_{ee} < 0$, the graduation probability for non-mathematics students decline. The policy change induces more students to take mathematics. As a result, the remaining students – those who do not take mathematics – have less preference for mathematics and thus a smaller graduation probability than the students who shift to mathematics. The lower curve in Figure 1 moves downwards.

Self-selection leads to the following change in the difference in graduation probability between mathematics and non-mathematics students

$$\begin{aligned} & \frac{\partial}{\partial \omega} \left[-\sigma_{e,\epsilon} \left[\frac{g(z_i)}{G(z_i)} + \frac{g(z_i)}{1 - G(z_i)} \right] \right] \\ &= \sigma_{e,\epsilon} \frac{g(z_i)}{G(z_i)(1 - G(z_i))} \left[z_i - 2g(z_i) \frac{G(z_i) - \frac{1}{2}}{(1 - G(z_i))G(z_i)} \right] \frac{\partial z_i}{\partial \omega}. \end{aligned}$$

For $z_i = 0$ the entity in the bracketed parenthesis is 0, and the policy change does not alter the difference in graduation probability. Under the assumption of $\sigma_{e,\epsilon} < 0$, the right hand side is negative for $z_i < 0$ and positive for $z_i > 0$. The expression is anti-symmetric, implying that the effect of a positive value of z_i is exactly offset by a negative value of z_i of the same numerical magnitude. For a given value of an explanatory variable as the GPA, the difference between graduation for the mathematics and the non-mathematics students thus tends to remain unaltered by the policy change. In Figure 1, both the upper and lower curve move downwards by about the same amount.

I have reached the following conclusions under the assumption that giving higher weight to mathematics in the admission criterion induces more students to take this subject: to the extent that (some of) the difference in graduation rates between mathematics and non-mathematics students is due to heterogeneous return from mathematics, the expected graduation rate for mathematics students will decrease whilst the expected graduation rate for non-mathematics students remains the same as before the policy change. The difference between the two groups of students in graduating from the programme will thus diminish. In contrast, to the extent that (some of) the difference in graduation between mathematics and non-mathematics students is due to a higher graduation probability for students with high relative preference for mathematics, the expected graduation rate for mathematics students will decrease, and the same holds for the expected graduation rate for non-mathematics students. The difference in graduation between the two groups of students will tend to be the same as before the policy change. The absence of both heterogeneous return and self-selection implies no change in the conditional expected graduation functions and the curves in Figure 1.

Hence, the incentive structure of the education system has consequences for the observed outcome of taking mathematics. This observation is relevant not only for differences in graduation rates in higher education but also for cross-country comparisons of the effect of mathematics on future outcomes such as earnings.

6 Conclusion

This article analyses admission policies of the higher education system and derives policy rules that maximise graduation rates. Introduction of optimal

admission results in increases in graduation rates, and the article presents approximate expressions for this increase.

Optimal admission will result in a high increase in graduation rate for a higher education programme if there are (1) large differences in initial graduation rates between various groups of students admitted to the programme, (2) a weak relation between graduation rates and general ability to study as measured by the GPA and (3) an equal composition of the intake of students from different groups.

The application of optimal admission to the social science programmes at the University of Copenhagen demonstrates that students with a strong secondary school background in mathematics ought to be the only ones admitted to psychology, and that they should have a huge preference in admission to the law programme but a small preference to the economics programme. The main reason for this result is the variation in the strength of the relation between graduation rates and general ability to study across the programmes.

In combination with the theory of this article, analogous empirical analysis form the basis of optimal admission rules. Optimal admission could be fine-tuned to single programmes at one university, or common rules can be applied to several programmes within or across universities.

Introduction of optimal admission is expected to alter the choice of subjects by secondary school students in such a way that they try to be better prepared to graduate from programmes in the higher education system. This altered choice is an indirect gain of a changed admission system. Optimal admission of students is a valid policy rule even when the entire difference in graduation rates between different groups of students is due to self-selection. In this case, whilst a direct gain of increased graduation rates exists, no indirect gain appears from applying a policy rule maximising graduation rates.

Most empirical results show that only a minor part of the estimated economic gain of one more year of education is due to self-selection (see e.g. the recent survey by Card (1999)). To the extent that self-selection also plays a minor role in the impact of secondary school subjects for understanding the content of programmes in the higher education system, there are derived gains of admission policies that maximise graduation rates. These derived gains have the potential of making whole cohorts of secondary school students better able to graduate with a higher education degree.

The theory developed in this article and the application of the theory in the empirical analysis demonstrates how to increase graduation rates and further the smooth transition of students through the higher educational sys-

tem. Furthermore, the article shows how an improved incentive structure in the secondary school system has the potential of making students better able to graduate from higher education. The application of an optimal admission policy for higher education is thus a means of increasing the skill level of the work force.

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Fig. 1. Transition from admission system with common threshold to optimal admission system

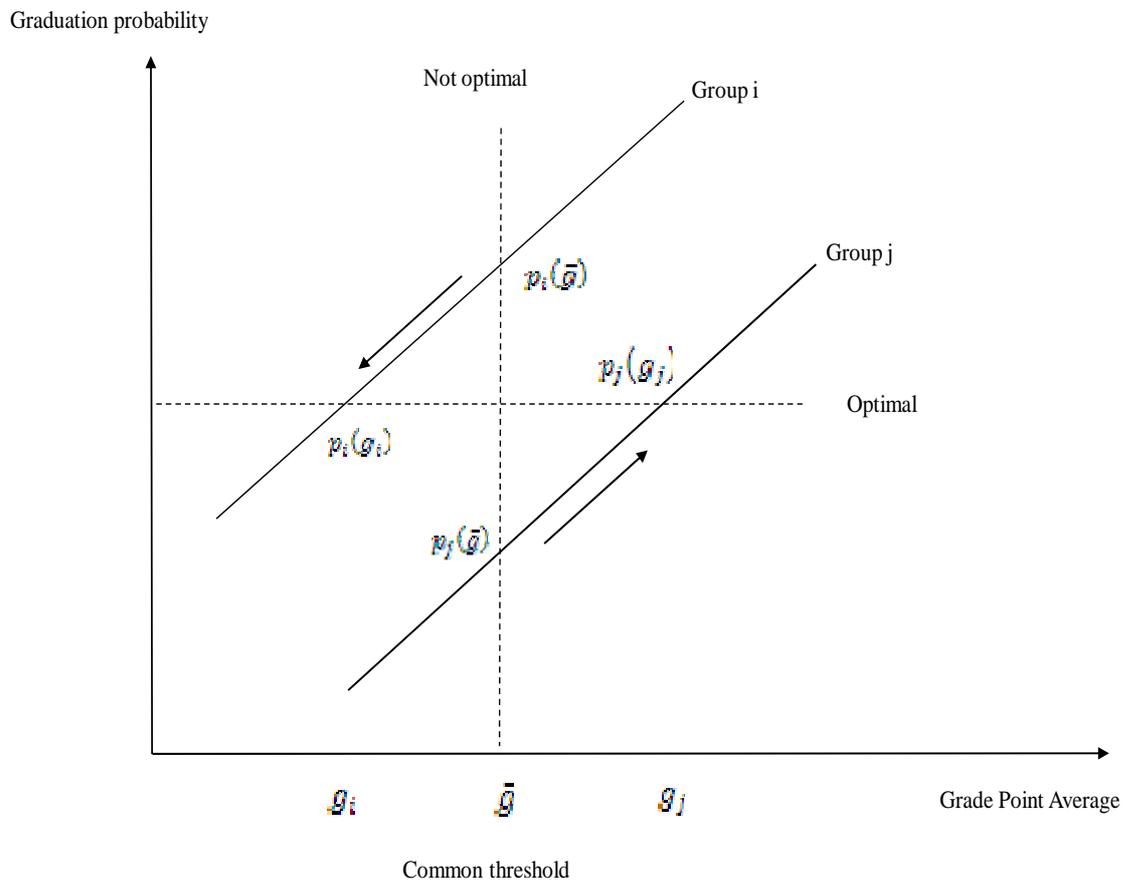


Fig.2. GPA for admitted students in social science

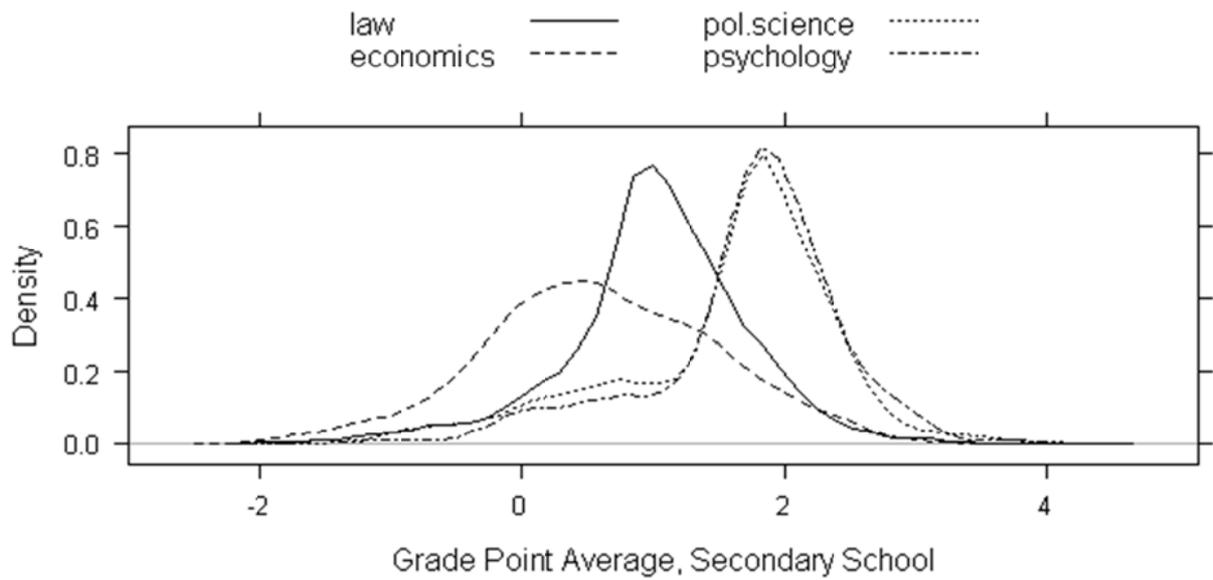
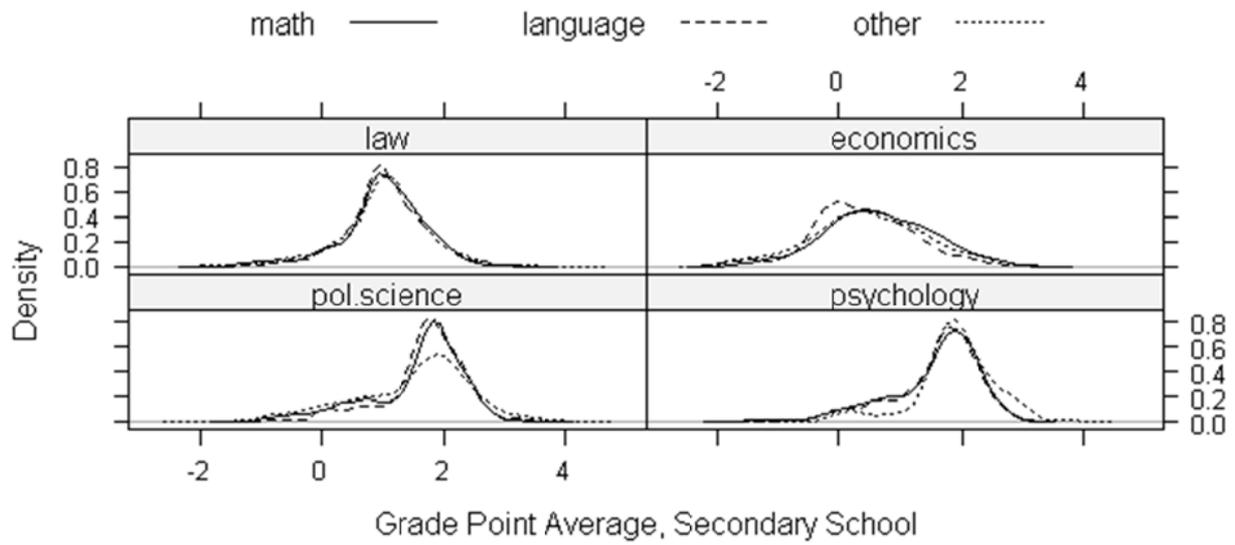
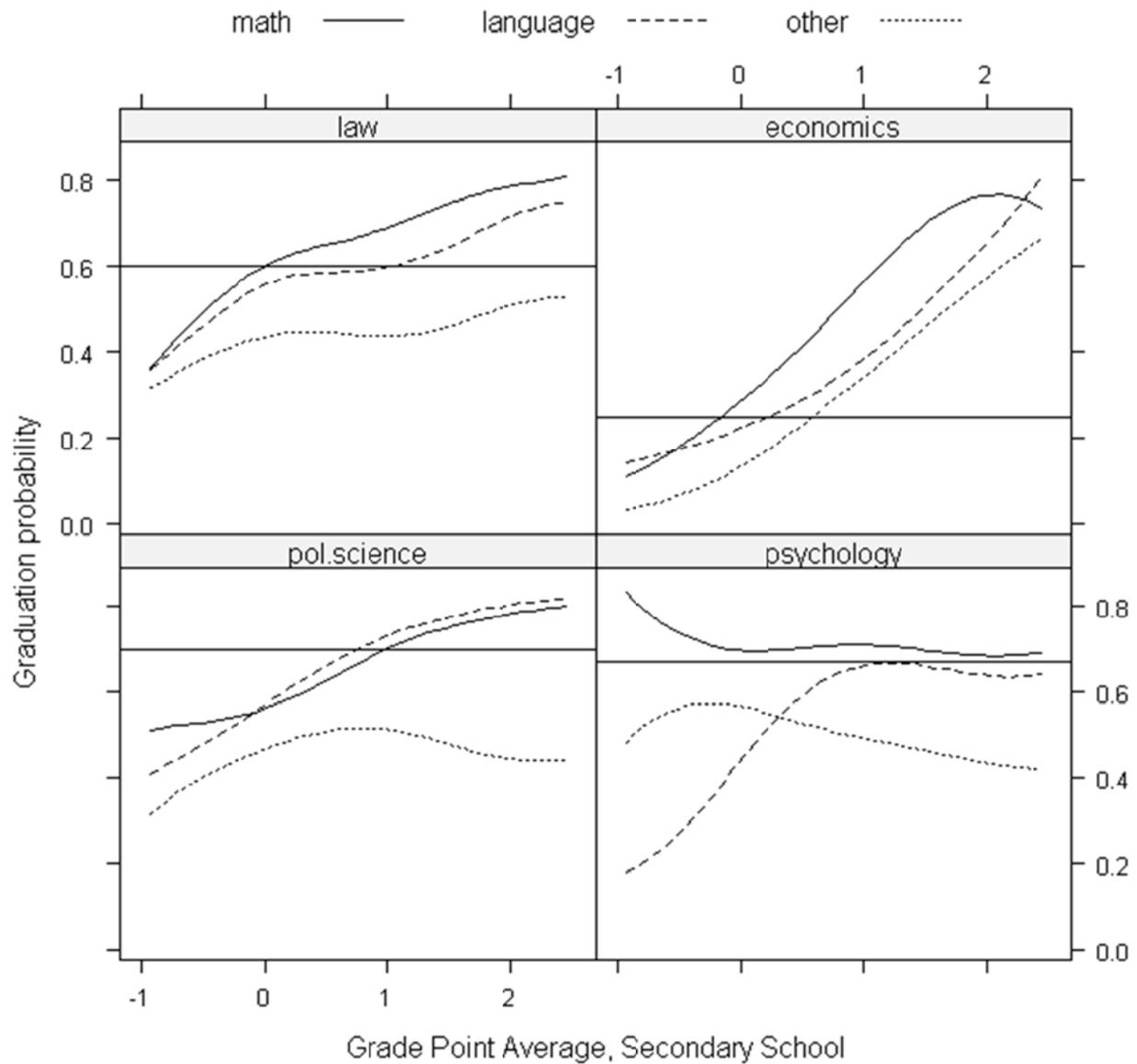


Fig. 3. Graduation probability and GPA in Social Science



Note: Optimal admission rules shown as horizontal lines

Fig. 4. Slope of Conditional Expected Graduation Functions

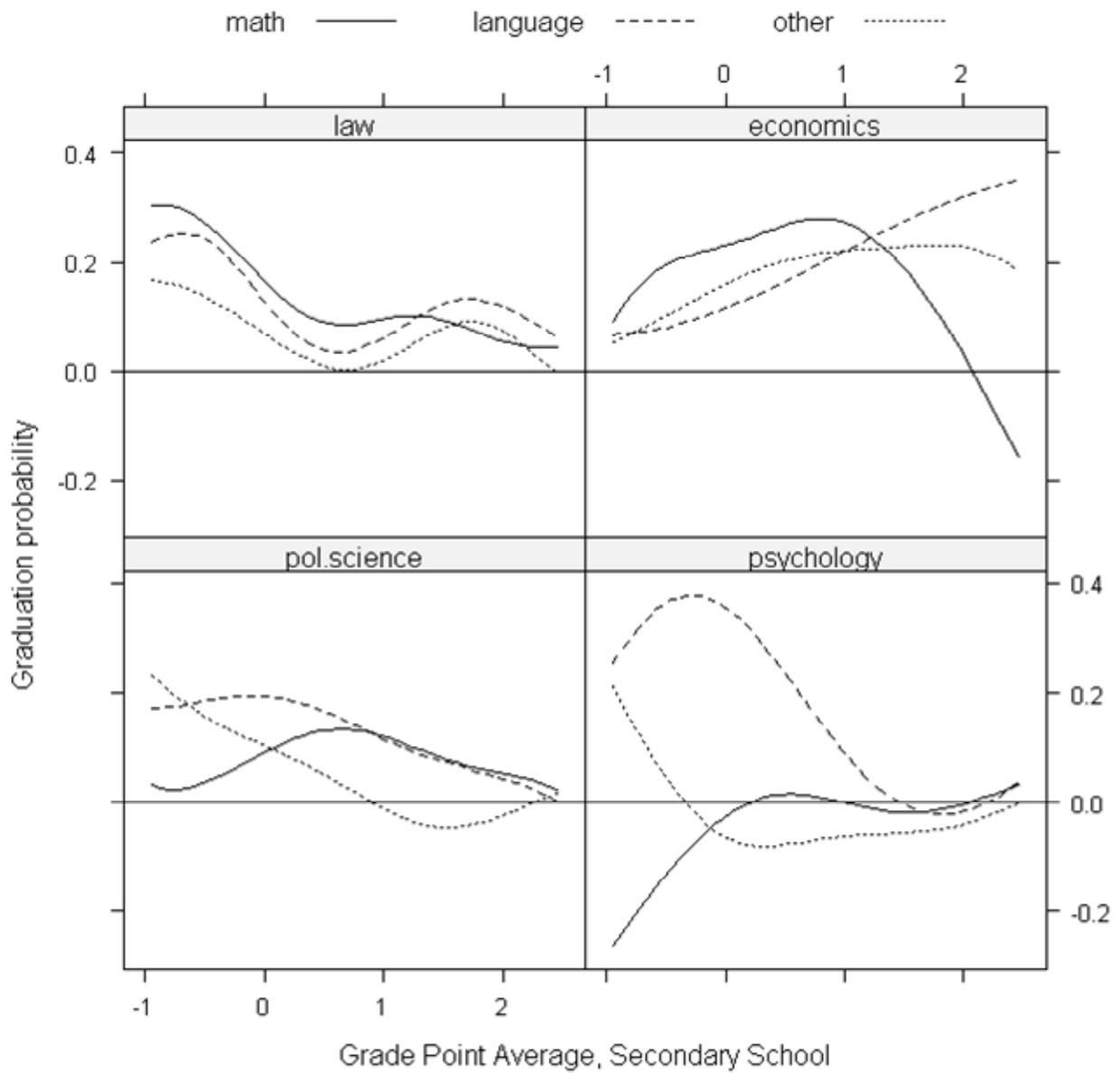


Table 1. *Sample statistics for three branches of secondary school and statistics for optimal admission.*

	Sample statistics			Common threshold ¹⁾		Threshold change ²⁾	Graduation change ³⁾	
	Share	Graduation rate	(Standard error)	Grad. rate	Slope		Points	Relative
Programme:								
Law								
Math	0.386	0.693	(0.011)	0.649	0.090	-0.460		
Language	0.329	0.616	(0.012)	0.586	0.040	0.539		
Other	0.285	0.450	(0.013)					
Total	1.000	0.598	(0.007)				0.062	0.104
Economics								
Math	0.685	0.466	(0.012)	0.284	0.229	-0.221		
Language	0.042	0.308	(0.042)	0.221	0.113	0.055		
Other	0.273	0.252	(0.016)	0.131	0.157	0.613		
Total	1.000	0.401	(0.009)				0.007	0.018
Pol. Science								
Math	0.455	0.735	(0.017)					
Language	0.299	0.766	(0.020)					
Other	0.246	0.460	(0.026)					
Total	1.000	0.677	(0.012)				0.068	0.100
Psychology								
Math	0.241	0.695	(0.027)					
Language	0.287	0.621	(0.026)					
Other	0.472	0.452	(0.021)					
Total	1.000	0.559	(0.014)				0.136	0.243
Total								
Math	0.457	0.610	(0.007)					
Language	0.245	0.629	(0.010)					
Other	0.298	0.404	(0.009)					
Total	1.000	0.553	(0.005)				0.057	0.103

Note: Standard errors in parenthesis. Admission to the University of Copenhagen Summer 1984 - Summer 2001. Total number of admitted students was 10418 distributed on 47.2% in law, 26.3% in economics, 14.7% in political science and 11.8% in psychology. Graduation with a bachelor degree within four years of study. ¹⁾ Graduation probabilities and slopes are measured at the common threshold. ²⁾ Changes in admission thresholds as a consequence of optimal admission. ³⁾ Percentage points increase in graduation rate as a consequence of optimal admission and increase relative to previous level of graduation.