Welfare effects of deterrence-motivated activation policy

The case of distinct activation-disutility

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Working Paper 09:2005

Working Paper

Socialforskningsinstituttet
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Welfare effects of deterrence-motivated activation policy: the case of distinct activation-disutility[1,2]

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Abstract

We investigate conditions for activation policy to be part of an optimal policy, when the motivation for activation is to deter people from collecting benefits. A benevolent government chooses a pure benefit programme and an activation programme and individuals self-select into programmes or work. We consider a distinct disutility for participating in activation programmes. One motivation for this approach is that the choice of concrete activation programmes may affect how people are exposed to activation-disutility. We describe a principle for the choice of optimal activation programmes, but find it hard to give real-world examples that meet the principle.

Theme: Labour market policy

Keywords: Workfare, Active labour market policy, Activation policy

JEL-Code: J65, J68

1. Introduction

[1] Financial assistance from the Danish Social Science Research Council is gratefully acknowledged.

[2] The paper is a companion paper to Rasmussen (2004). That paper is generally more detailed.

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We investigate theoretically whether use of ‘activation policy’ (i.e. the policy of making social benefits conditioned of some sort of work-requirement, also phrased ‘workfare’ or ‘active labour market policy’) can be part of an optimal social policy. We limit the discussion to the deterrence effect or motivation effect of activation policy, i.e. the effect caused by some people choosing employment rather than public benefits because activation programmes implies disutility analogous to disutility related to ordinary employment. We consider a simple model with a benevolent government. The government chooses a *pure benefit* programme and an *activation* programme. The first consists simply of a benefit for the unemployed and the second of a (higher) benefit combined with a requirement of ‘activation’, i.e. some sort of effort carried out by the participants. Individuals self-select into programmes or ordinary work. Individuals know their own characteristics, whereas the government only knows the distribution of characteristics across the population.

Other papers in the field, i.e. Besley and Coate (1992, 1995), Brett (1998), Cuff (2001), Thustrup Kreiner and Tranæs (2003), Blackorby and Beaudry (1997) and the companion paper Rasmussen (2004) consider private-knowledge parameters to be the wage rate and/or a parameter for disutility in work as well as activation. The special feature of this paper is that we consider and a distinct private-knowledge disutility parameter for activation. It is clear from Rasmussen (2004) that two (or more) private-knowledge parameters are necessary for deterrence-motivated activation policy to be part of optimal social policy. In this paper, the second private-knowledge parameter is disutility related to ordinary work in one version of the model, and the wage rate in another version.

There are two motivations for considering a distinct disutility in activation. The first is that is realistic: surely for many people and for many types of activation programmes, work an activation programme is not the same as work in an ordinary job. The second motivation is that the model may be used to give guidelines for the choice of activation programmes, because the government might be able to affect how people are exposed to disutility in activation via the concrete choice of the programmes. As in the companion paper Rasmussen (2004), the distribution of private-knowledge parameters is closely related to optimal use of activation programmes. A necessary condition for an activation programme to be part of optimal policy is that the ‘adverse labour supply effects’ from social benefits should be low among the group who potentially participate in activation (see Rasmussen (2004) or below). One principle for choice of activation programmes is therefore to search for activation programmes that meet this condition. We find it however difficult to imagine how the principle should be used in the real world.

As mentioned, the paper is a companion to Rasmussen (2004). That paper is longer and more detailed in many aspects and the reader might wish to read that paper first. This paper might be seen as an appendix to the other paper.

The paper is organised as follows. In section 2, we set up the model with disutility parameters related to activation and related to work as the two private-knowledge parameters. We write a necessary condition for activation programmes to be part of optimal policy in section 3 (identical to the condition in the companion paper Rasmussen (2004)) and discuss a principle for choice of programmes. In two subsections we investigate examples. In a short section 4 we show that a model with the wage rate as a private knowledge parameter in place of work-disutility has a structure very similar to the model studied in the previous sections. Section 5 concludes.

2. A model with a distinct disutility parameter for activation and for ordinary work.

Population and private knowledge: Private-knowledge parameters are disutility related to work, d, and disutility related to the work-requirement in activation, g. We assume that $g, d \geq 0$ and that the joint density function is f. An individual is denoted (g, d). Each individual knows her own disutility parameters. The government only knows the distribution of characteristics across individuals.

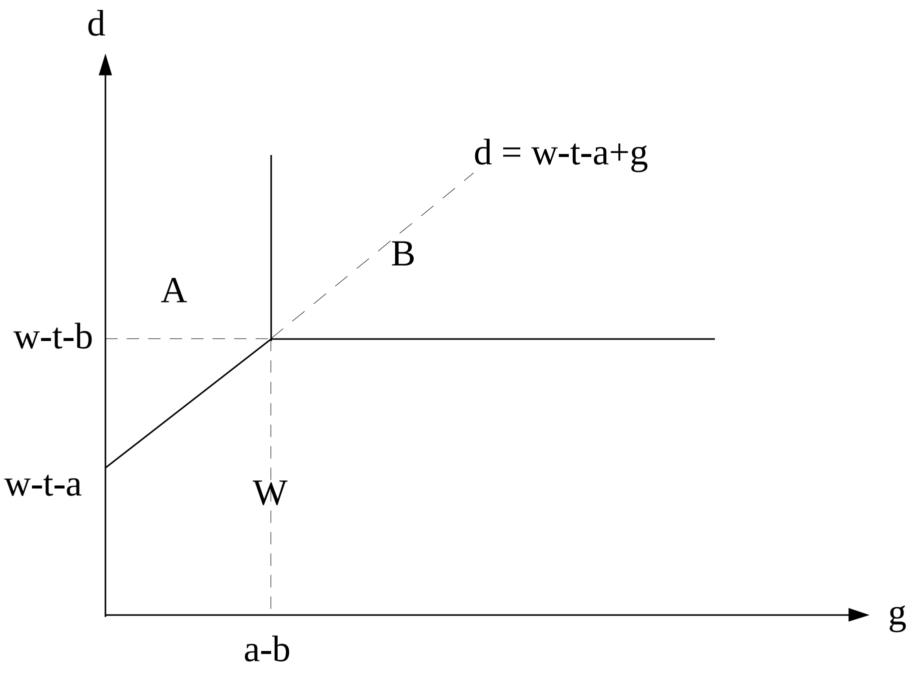
Individuals' utility in various states: An individual derives utility from income and disutility from work or activation. On the basis of the obtainable level of 'income minus disutility' in the three 'states', *work*, *pure benefit*, and *activation*, the individual chooses a state. If the individual works, she supply one unit of labour. From work, she obtains a wage, w (the wage is the same for all individuals), disutility, d, and pay the lump sum tax rate t. In the activation programme, the benefit rate net of tax is a and the disutility is g. In the pure benefit programme there is no disutility and the benefit rate net of tax is b.

We denote by $B, A,$ and W the sets of individuals who choose pure benefit, activation, or work. The distribution of individuals on states as function of disutility parameters, and politically determined variables, is

$$
\begin{aligned}
&W = \{(g,d) \mid w-t-d \geq b \text{ and } w-t-d \geq a-g\} \\
&= \int_{g=0}^{a-b} \int_{d=0}^{w-t-a+g} f(g,d)\mathrm{d}d\mathrm{d}g + \int_{g=a-b}^{\infty} \int_{d=0}^{w-t-b} f(g,d)\mathrm{d}d\mathrm{d}g \\
&A = \{(g,d) \mid a-g > b \text{ and } a - g > w-t-d\} = \int_{g=0}^{a-b} \int_{d=w-t-a+g}^{\infty} f(g,d)\mathrm{d}d\mathrm{d}g \\
&B = \{(g,d) \mid b \geq a-g \text{ and } b > w-t-d\} = \int_{g=a-b}^{\infty} \int_{d=w-t-b}^{\infty} f(g,d)\mathrm{d}d\mathrm{d}g
\end{aligned}
\tag{1}
$$

Figure 1 illustrates the sets of individuals.

Figure 1. Distribution of individuals on states as function of policy variables



In drawing figure 1 we implicitly assume $a, b, t \geq 0$, and $a > b$. Negative benefits are excluded because the model implicitly assumes a fourth 'programme', namely a programme with no benefits and no work (e.g. home working wives) that is preferable to a programme with negative benefits. In optimum negative benefits would therefore never occur. Since taxes finance the costs of the programmes, the tax rate will also be positive. Furthermore, $a < b$ would imply that no individual would choose activation.

Government's problem: The government's criterion function depends on 'income minus disutility' for each individual. Each individual contributes to the criterion function through the increasing, concave function u, $u:[0,\infty)\to[0,\infty)$. The criterion function is

$$
\begin{aligned}
V &= \int_{(d,g)\in W} u(w-t-d)\,\mathrm{d}(d,g) + \int_{(d,g)\in A} u(a-g)\mathrm{d}(d,g) + \int_{(d,g)\in B} u(b)\,\mathrm{d}(d,g) \\
&= \int_{g=0}^{a-b} \int_{d=0}^{w-t-a+g} u(w-t-d)\,f(g,d)\mathrm{d}d\mathrm{d}g + \int_{g=a-b}^{\infty} \int_{d=0}^{w-t-b} u(w-t-d)\,f(g,d)\mathrm{d}d\mathrm{d}g \\
&+ \int_{g=0}^{a-b} \int_{d=w-t-a+g}^{\infty} u(a-g)\,f(g,d)\mathrm{d}d\mathrm{d}g \\
&+ \int_{g=a-b}^{\infty} \int_{d=w-t-b}^{\infty} u(b)\,f(g,d)\mathrm{d}d\mathrm{d}g
\end{aligned}
\tag{2}
$$

The government's budget constraint is

$$F = Wt - Aa - Bb = 0 \tag{3}$$

The government's problem is to maximize V subject to the budget constraint. The policy variables are the benefit rates and the tax rate.

3. Conditions for solution of government's problem

We restate in proposition 1 the necessary condition for activation policy to be part of optimal policy, which was given in Rasmussen (2004).

Proposition 1. A necessary condition for activation policy to be part of optimal policy is that

$$\left|\frac{\partial W/\partial a}{A}\right| < \left|\frac{\partial W/\partial b}{B}\right| \tag{4}$$

holds at optimal values of $a, b,$ and t. •

Proof. The Lagrange function is

$$L = V - \lambda F$$

Derivatives of the budget are (using $\frac{\partial B}{\partial t} = -\frac{\partial W}{\partial t} - \frac{\partial A}{\partial t}$ and similarly for a and b)

$$F_t = (b+t)\frac{\partial W}{\partial t} - (a-b)\frac{\partial A}{\partial t} + W$$

and analogously for a and b. We denote the marginal utility of participants of the pure benefit programme and averages marginal utility of activation participants and people in work as follows

$$u'(b) = \partial u(b) / \partial b$$

$$\bar{u}'(a-d) = \frac{1}{A} \int_{(w,d)\in A} u'(a-d) f(w,d) \mathrm{d}(w,d) \quad (5)$$

$$\bar{u}'(w-t-d) = \frac{1}{W} \int_{(w,d)\in W} u'(w-t-d) f(w,d) \mathrm{d}(w,d)$$

In Rasmussen (2004) we show that

$$\frac{\partial V}{\partial b} = u'(b), \frac{\partial V}{\partial a} = \bar{u}'(a-d), \frac{\partial V}{\partial t} = -\bar{u}'(w-t-d). \quad (6)$$

(that is, the effect on V from individuals who change state (e.g. from work to pure benefits) do not affect V.)

The Lagrange conditions for an interior solution are (an interior solution implies $a > b > 0,$ and $A, B > 0$)

$$L_a = \bar{u}'(a-g)A - \lambda((b+t)\frac{\partial W}{\partial a} - (a-b)\frac{\partial A}{\partial a} - A) = 0$$
$$L_b = \bar{u}'(b)B - \lambda((b+t)\frac{\partial W}{\partial b} - (a-b)\frac{\partial A}{\partial b} - B) = 0 \qquad (7)$$
$$L_t = -\bar{u}'(w-t-d)W - \lambda((b+t)\frac{\partial W}{\partial t} - (a-b)\frac{\partial A}{\partial t} - W) = 0$$
$$F = Wt - aA - bB = 0$$

Rewrite the first two conditions as

$$\frac{L_a}{A} = \bar{u}'(a-g) - \lambda\left[(b+t)\frac{\partial W/\partial a}{A} - (a-b)\frac{\partial A/\partial a}{A} - 1\right] = 0$$
$$\frac{L_b}{B} = \bar{u}'(b) - \lambda\left[(b+t)\frac{\partial W/\partial b}{A} - (a-b)\frac{\partial A/\partial b}{A} - 1\right] = 0 \qquad (8)$$

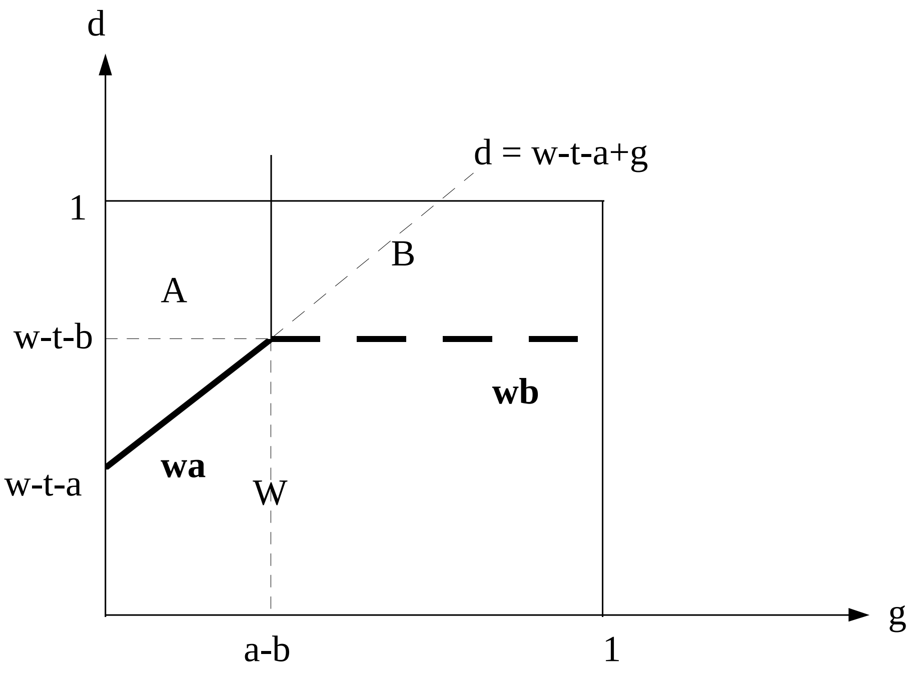
It is clear that $\lambda < 0$ in an optimum and easy to show that $\frac{\partial W}{\partial a}, \frac{\partial W}{\partial b} < 0,\ \frac{\partial A}{\partial b} < 0 < \frac{\partial A}{\partial a}$ (see the appendix). Also, $u'(b) > \bar{u}'(a-g)$: this follows from self-selection and concavity of u. Condition (4) must therefore hold if both equations are to be fulfilled.

Next, consider the candidates for border solutions. The candidates are i) no programmes open ($a = b = 0$), ii) only the pure benefit programme open ($a = b > 0$), and iii) only the activation programme open ($a > b = 0$). Case iii) implies $L_b \le 0$ and $L_a = 0$. The arguments above use $B > 0$ to show that condition (4) is necessary for $L_a \ge L_b$. If we refine the set B to include individuals who receive a zero benefit $b = 0$ (i.e. home-working wives), $B > 0$ still holds, and condition (4) is therefore still a necessary condition for activation policy to be optimal. Case ii) applies if (4) does not hold. Case iii) turns out not to be possible in this specific model. This is because benefit rates equal to zero implies $t = 0$ since the government uses the tax revenue only to finance the social programmes. Consequently, $a = b = t = 0$. Using this together with derivates of the Lagrange implies $u'(0) < \bar{u}'(w - d)$ which is not true.

•

The measure $\frac{\partial W}{\partial a}$ is the bold line in figure 2 below, and $\frac{\partial W}{\partial b}$ is the bold dotted line (in the figure we furthermore assume that $d, g \in [0,1]$).

Figure 2. Distribution of individuals on states as function of policy variables



Condition (8) suggest the definition of three 'effects' (as in Rasmussen (2004))

- A 'targeting effect', $\bar{u}'(a-g) < u'(b)$: the activation programme benefits those who are relatively well off.
- A 'direct budget effect', $(a-b)\frac{\partial A}{\partial a} > 0 > (a-b)\frac{\partial A}{\partial b}$: activation benefits are simply more expensive than pure benefits for the government.
- The 'adverse labour effect' $\frac{\partial W / \partial a}{A}$ compared to $\frac{\partial W / \partial b}{B}$.

Condition (4) as a guide to choice of activation programmes

Perhaps condition (4) may be use as a guide to choosing activation programmes. The government does not know the characteristics of each individual but it is assumed to know the distribution of characteristics. Different types of programmes will surely be related with different types of disutility. To choose an activation programme that could be used as part of optimal social policy, the government should search for a programme such that many potential participants have very high disutility related to ordinary work (in figure 2, the density near $d = 1$ and $g = 0$ is high) and there are few participants who are almost indifferent between work and activation (the density near $\boldsymbol{wa}$ is low). For (4) to be fulfilled, the opposite should hold for the pure benefit programme, i.e. a low density near $d = 1$ and $g > a - b$ and a high density near $\boldsymbol{wb}$. It is however not immediately easy to come up with real-world ideas for a programme with such characteristics.

3.1. A uniform distribution

Assume that d and g are independent and uniformly distributed on the domain $[0,1]$. Suppose further that $w < 1$. This assumption implies that individuals with the highest disutility would choose not to work in the absence of any social programmes.

The sets of individuals become (see figure 2)

$$
\begin{aligned}
W &= \{(d,g) \mid w-t-d \geq b \text{ and } w-t-d \geq a-g\} = (w-t-b) - 0.5(a-b)^2 \\
A &= \{(d,g) \mid a-g > w-t-d \text{ and } a-g > b\} = \left(1-(w-t-b)\right)(a-b) + 0.5(a-b)^2 \qquad (9) \\
B &= \{(d,g) \mid b > w-t-d \text{ and } b \geq a-g\} = \left(1-(w-t-b)\right)\left(1-(a-b)\right)
\end{aligned}
$$

Proposition 2. Condition (4) is fulfilled in the case of uniform, independent distribution. •

Proof. We get $\partial W/\partial b = -1 + (a-b)$, $\partial W/\partial a = -(a-b)$ and therefore

$$
-\frac{\partial W/\partial b}{B} = \frac{1}{1-(w-t-b)} > \frac{(a-b)}{\left(1-(w-t-b)\right)(a-b) + 0.5(a-b)^2} = -\frac{\partial W/\partial a}{A} \quad \bullet
$$

Proposition 3. In the case of a uniform independent distribution

$$
(b+t)\frac{\partial W/\partial b}{B} - (a-b)\frac{\partial A/\partial b}{B} < (b+t)\frac{\partial W/\partial a}{A} - (a-b)\frac{\partial A/\partial a}{A} \quad \bullet
$$

The proposition implies that activation policy is not part of optimal social policy because the sum of the labour supply effect and the direct budget effect is large for the activation programme compared to the pure benefit programme.

Proof. Put

$$
\begin{aligned}
\beta &= -(b+t)\frac{\partial W/\partial b}{B} + (a-b)\frac{\partial A/\partial b}{B} \\
&= -(b+t)\frac{1-(a-b)}{\left(1-(w-t-b)\right)\left(1-(a-b)\right)} + (a-b)\frac{\left(1-(w-t-b)\right)-(a-b)}{\left(1-(w-t-b)\right)\left(1-(a-b)\right)}
\end{aligned}
$$

and

$$\alpha = -(b+t)\frac{\partial W/\partial a}{A} + (a-b)\frac{\partial A/\partial a}{A}$$

$$> -\frac{(a-b)(b+t) + (a-b)\big(1-(w-t-b)+(a-b)\big)}{\big(1-(w-t-a)\big)(a-b)}$$

$$= -\frac{b+t}{1-(w-t-a)} - 1 = -\frac{(b+t)+\big(1-(w-t-a)\big)}{1-(w-t-a)}$$

We show that $\alpha > \beta$.

$$\alpha - \beta > 0$$

$$\Leftrightarrow$$

$$\Big(\big(1-(w-t-b)\big)\big(1-(a-b)\big)\Big)\Big((b+t)-\big(1-(w-t-a)\big)\Big)$$

$$-\Big((b+t)\big(1-(a-b)\big)-(a-b)\big(1-(w-t-b)\big)\Big)\big(1-(w-t-a)\big) > 0$$

$$\Leftrightarrow$$

$$(a-b)\big(1-(w-t-b)\big)\big(1-(w-t-a)\big)+(1-w)\big(1-(a-b)\big)\big(1-(w-t-a)\big)$$

$$+\big(1-(w-t-b)\big)\big(1-(a-b)\big)(b+t) > 0 \quad \text{(which is true because } w<1\text{)}$$

3.2. Many activation programmes

We discussed above a guide for choice of activation programmes. In this subsection, we will be a little more precise by assuming the existence of N potential activation programmes each of which the government considers to open.

A participant in activation programme i obtains utility $a_i - g_i$. The disutility parameter g_i is a private-knowledge disutility parameter for programme i. There are therefore N private-knowledge parameters for activation plus the disutility parameter d related to ordinary work. An individual is denoted $(g_1, g_2, ..., g_N, d)$.

Let A_i denote the set of individuals who participate in activation programme i. The set is (ignoring ties)

$$A_i = \{(g_1, g_2, ..., g_N, d) \mid a_i - g_i > a_j - g_j \text{ for } j \neq i, a_i - g_i > b, a_i - g_i > w - t - d\} \qquad (10)$$

Let 1_i denote an indicator-function that takes the value 1 if programme i is open and 0 if the programme is not open. By 'open' we mean that A_i is not empty. We can consider a problem for the government analogous to above and derive first order conditions as

$$u'(b)B = -\lambda\left[(b+t)\frac{\partial W}{\partial b} - \sum_{i\in\{1,...,N\}} 1_i (a_i - b)\frac{\partial A_i}{\partial b} - B\right]$$

$$\bar{u}'(a_i - g_i)A_i = -\lambda\left[(b+t)\frac{\partial W}{\partial a_i} - (a_i - b)\frac{\partial A_i}{\partial a_i} - \sum_{j\in\{1,...,N\}\backslash i} 1_j (a_j - b)\frac{\partial A_j}{\partial a_i} - A_i\right] \qquad (11)$$

if i is open

$$\bar{u}'(a_i - g_i) \leq -\lambda\left[(b+t)\frac{\partial W}{\partial a_i} - (a_i - b)\frac{\partial A_i}{\partial a_i} - \sum_{j\in\{1,...,N\}\backslash i} 1_j (a_j - b)\frac{\partial A_j}{\partial a_i}\right]$$

if i is not open[4]

Individuals move from one open activation programme to another. A particular open activation programme may therefore have low costs if many move from other expensive activation programmes, i.e. if the term $\sum_{j\in\{1,...,N\}\backslash i} 1_j (a_j - b)\frac{\partial A_j}{\partial a_i}$ is large.

---

[4] For a programme that is not open, we define $\bar{u}'(a_i - g_i)$ in the following way. Let $\hat{g}_i = \text{Min}\{g_i\}$ i.e. the disutility for the individual with lowest disutility for the programme. Let $((\hat{g}_i)_{i=1,...N}, \hat{d})$ denote that individual's private-knowledge parameters. Set a_i such that $a_i - \hat{g}_i = \text{Max}\left\{(a_j - \hat{g}_j)_{j\neq i}, (w - t - \hat{d})\right\}$ and finally put $\bar{u}'(a_i - g_i) = u'(a_i - \hat{g}_i)$. To sum up, we assume the benefit rate for a closed programme is set such that the individual who would obtain the highest utility from the programme is just indifferent between the closed programme and her preferred programme. The marginal utility for the programme is that individual's marginal utility.

Given that some activation programmes are open, a new programme might therefore appear attractive according to (11). However, a necessary condition for any activation programme to be open similar to (4) still exists. To see this, imagine a coordinated increase of $a_1,...,a_N$ so that $\frac{\partial A_j}{\partial a_i}$'s are zero (programmes do not steal participants from each other). In this way, a version of (11) can be written which is very similar to (8), i.e. where the terms $\sum_{j\in\{1,...,N\}\setminus i} 1_j(a_j - b)\frac{\partial A_j}{\partial a_i}$ disappear from (11).

4. Activation-disutility and wage rate as private-knowledge parameters

In this section, we assume that each individual has a wage rate, w, which is particular for her, but the disutility parameter related to ordinary work, d, is constant across individuals. The roles of wage and work-disutility are therefore reversed compared to the previous sections. Put $d = 0$. An individual is denoted (w, g). The approach is otherwise as in the previous section.

The sets of individuals are

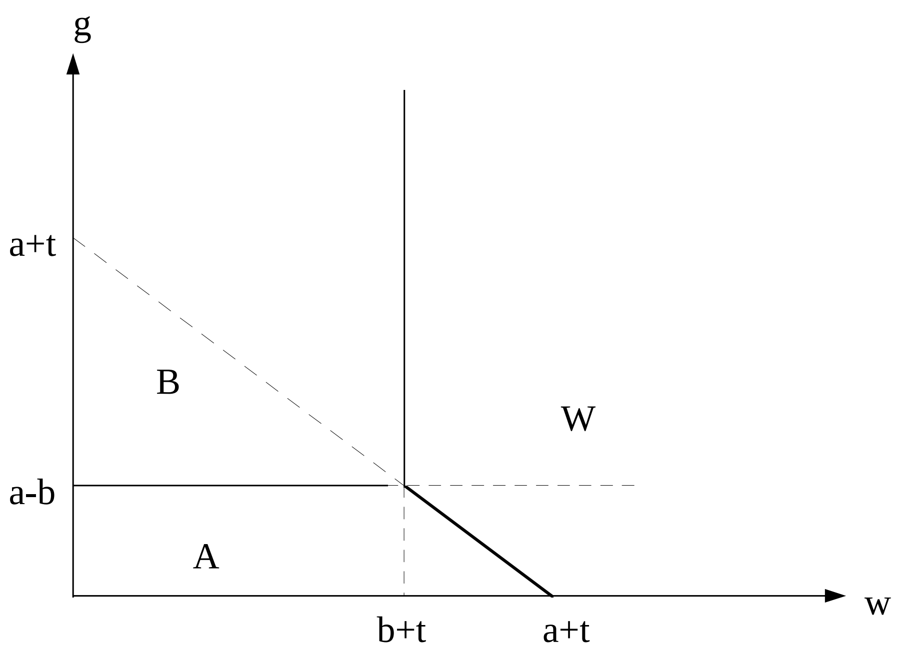
$$A = \{(w,g) \mid a - g > w - t - d \text{ and } a - g > b\} = \int_{g=0}^{a-b} \int_{w=0}^{a+t-g} f(w,g)\mathrm{d}w\mathrm{d}g$$

$$B = \{(w,g) \mid b > w - t - d \text{ and } b \geq a - g\} = \int_{w=0}^{b+t} \int_{g=a-b}^{\infty} f(w,g)\mathrm{d}g\mathrm{d}w \qquad (12)$$

$$W = \{(w,g) \mid w - t - d \geq b \text{ and } w - t - d \geq a - g\} = 1 - A - B$$

Figure 3 shows the sets

Figure 3. Distribution of individuals across states



It is straightforward to derive a necessary condition for an activation programme to be part of optimal social policy exactly as condition (4) in the previous case.

5. Conclusion

The paper uses the approach in the companion paper Rasmussen (2004) to consider a special case of the models that investigate welfare and activation policy: namely the case with disutility parameters in activation different from disutility in ordinary work. This is in contrast to other contributions in the field that consider wage and work-disutility as private-knowledge parameters. There are at least two reasons for this approach: first, it is a realistic variant, and, second, it opens for a discussion of how the authority should choose activation programmes. We discuss a general, abstract guide for choice of activation programmes. This guide appears however difficult to implement in the real world.

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Appendix

Rewrite (1)

$$W = \{(g,d) \mid w-t-d \geq b \text{ and } w-t-d \geq a-g\}$$
$$= \int_{g=0}^{a-b} \int_{d=0}^{w-t-a+g} f(g,d)\mathrm{d}d\mathrm{d}g + \int_{g=a-b}^{\infty} \int_{d=0}^{w-t-b} f(g,d)\mathrm{d}d\mathrm{d}g$$

$$A = \{(g,d) \mid a-g > b \text{ and } a - g > w-t-d\} = \int_{g=0}^{a-b} \int_{d=w-t-a+g}^{\infty} f(g,d)\mathrm{d}d\mathrm{d}g$$

$$B = \{(g,d) \mid b \geq a-g \text{ and } b > w-t-d\} = \int_{g=a-b}^{\infty} \int_{d=w-t-b}^{\infty} f(g,d)\mathrm{d}d\mathrm{d}g$$

Relevant derivatives are

$$\frac{\partial A}{\partial b} = -\int_{d=w-t-b}^{\infty} f(a-b,d)\mathrm{d}d$$

$$\frac{\partial B}{\partial b} = \int_{g=a-b}^{\infty} f(g,w-t-b)\mathrm{d}g + \int_{d=w-t-b}^{\infty} f(a-b,d)\mathrm{d}d$$

$$\frac{\partial W}{\partial b} = -\frac{\partial A}{\partial b} - \frac{\partial B}{\partial b} = -\int_{g=a-b}^{\infty} f(g,w-t-b)\mathrm{d}g$$

lkj

$$\frac{\partial A}{\partial a} = \int_{d=w-t-b}^{\infty} f(a-b,d)\mathrm{d}d + \int_{g=0}^{a-b} f(g,w-t-a+g)\mathrm{d}g$$

$$\frac{\partial B}{\partial a} = -\int_{d=w-t-b}^{\infty} f(a-b,d)\mathrm{d}d$$

$$\frac{\partial W}{\partial b} = -\int_{g=0}^{a-b} f(g,w-t-a+g)\mathrm{d}g$$